Mansoura University Faculty of Engineering

Common

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Course Title: Differential Equations

Course Code: Math(106)

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Date: 1-1-15 Time: 2Hours Final Exam Full Mark: 50

[1]-(a) Find the extremum of the function

$$f(x,y) = \frac{x^5}{5} - x + \cos y$$
 [5 pts]

(b) If $\phi(x, y, z) = x^2 y z^3$ and $A = x z i - y^2 j + 2 x^2 y k$ find

$$\nabla \phi$$
 $\nabla \cdot A$ Curl (ϕA) [5 pts]

(c) Evaluate

$$\int \int_A e^{x/y} \, dy \, dx$$

where A is bounded by parabola $y^2 = x$ and the straight lines x = 0, y = 1. [5 pts]

(d) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2 + y^2} \, dy \, dx. \qquad \int_1^3 \int_x^{x^2} \int_0^{\ln z} x \, e^y \, dy \, dz \, dx \qquad [10 \, \text{pts}]$$

(e) If $z = xy + xf\left(\frac{y}{x}\right)$

Find

$$x z_x + y z_y$$
 [5 pts]

[2]-(a) Find the orthogonal trajectories of

$$x^2 + 2xy - y^2 + 4x - 4y = C$$
 [5 pts]

(b) Solve by any method

1.
$$(\sin x \cosh y) dx - (\cos x \sinh y) dy = 0$$
 [5 pts]

2.
$$x^3y''' - 4x^2y'' + 8xy' - 8y = 4 \ln x$$
, $x > 0$. [5 pts]

(c) Find a linear homogeneous constant-coefficient equation with the given general solution.

$$y(x) = (A + Bx + Cx^{2}) e^{2x}$$
 [5 pts]