

[1]-(a) Find the extremum of the function

$$f(x, y) = \frac{x^5}{5} - x + \cos y \quad [5 \text{ pts}]$$

(b) If $\phi(x, y, z) = x^2 y z^3$ and $A = x z \mathbf{i} - y^2 \mathbf{j} + 2 x^2 y \mathbf{k}$
 find

$$\nabla \phi \quad \nabla \cdot A \quad \text{Curl}(\phi A) \quad [5 \text{ pts}]$$

(c) Evaluate

$$\int \int_A e^{x/y} dy dx$$

where A is bounded by parabola $y^2 = x$ and the straight lines $x = 0$, $y = 1$. [5 pts]

(d) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{x^2 + y^2} dy dx, \quad \int_1^3 \int_x^{x^2} \int_0^{\ln z} x e^y dy dz dx \quad [10 \text{ pts}]$$

(e) If $z = x y + x f\left(\frac{y}{x}\right)$

Find

$$x z_x + y z_y \quad [5 \text{ pts}]$$

[2]-(a) Find the orthogonal trajectories of

$$x^2 + 2xy - y^2 + 4x - 4y = C \quad [5 \text{ pts}]$$

(b) Solve by any method

$$1. (\sin x \cosh y) dx - (\cos x \sinh y) dy = 0 \quad [5 \text{ pts}]$$

$$2. x^3 y''' - 4x^2 y'' + 8xy' - 8y = 4 \ln x, \quad x > 0. \quad [5 \text{ pts}]$$

(c) Find a linear homogeneous constant-coefficient equation with the given general solution.

$$y(x) = (A + Bx + Cx^2) e^{2x} \quad [5 \text{ pts}]$$